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1. Define relative risk
2. Odds ratio
3. Confidence intervals
Motivation

- Consider a randomized trial where 40 subjects were randomized (20 each) to two drugs with the same active ingredient but different expedients.
- Consider counting the number of subjects with side effects for each drug.

<table>
<thead>
<tr>
<th>Side Effects</th>
<th>None</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drug A</td>
<td>11</td>
<td>9</td>
</tr>
<tr>
<td>Drug B</td>
<td>5</td>
<td>15</td>
</tr>
<tr>
<td>Total</td>
<td>16</td>
<td>14</td>
</tr>
</tbody>
</table>
Comparing two binomials

- Let $X \sim \text{Binomial}(n_1, p_1)$ and $\hat{p}_1 = X/n_1$
- Let $Y \sim \text{Binomial}(n_2, p_2)$ and $\hat{p}_2 = Y/n_2$
- We also use the following notation:

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$n_{11}$ = $X$</td>
<td>$n_{12}$ = $n_1 - X$</td>
<td>$n_1$ = $n_{1+}$</td>
</tr>
<tr>
<td>$n_{21}$ = $Y$</td>
<td>$n_{22}$ = $n_2 - Y$</td>
<td>$n_2$ = $n_{2+}$</td>
</tr>
<tr>
<td>$n_{2+}$</td>
<td>$n_{+2}$</td>
<td></td>
</tr>
</tbody>
</table>
• Last time, we considered the absolute change in the proportions, what about relative changes?
• Relative changes are often of more interest than absolute, e.g. when both proportions are small
• The **relative risk** is defined as \( \frac{p_1}{p_2} \)
• The natural estimator for the relative risk is

\[
\hat{RR} = \frac{\hat{p}_1}{\hat{p}_2} = \frac{X/n_1}{Y/n_2}
\]

• The standard error for log \( \hat{RR} \) is

\[
\hat{SE}_{\log \hat{RR}} = \left( \frac{1 - p_1}{p_1 n_1} + \frac{1 - p_2}{p_2 n_2} \right)^{1/2}
\]

• Exponentiate the resulting interval to get an interval for the RR
The odds ratio is defined as

\[
\frac{\text{Odds of SE Drug A}}{\text{Odds of SE Drug B}} = \frac{p_1/(1 - p_1)}{p_2/(1 - p_2)} = \frac{p_1(1 - p_2)}{p_2(1 - p_1)}
\]

The sample odds ratio simply plugs in the estimates for \(p_1\) and \(p_2\), this works out to have a convenient form

\[
\hat{\text{OR}} = \frac{\hat{p}_1/(1 - \hat{p}_1)}{\hat{p}_2/(1 - \hat{p}_2)} = \frac{n_{11}n_{22}}{n_{12}n_{21}}
\]

(cross product ratio)

The standard error for \(\log \hat{\text{OR}}\) is

\[
\hat{SE}_{\log \hat{\text{OR}}} = \sqrt{\frac{1}{n_{11}} + \frac{1}{n_{12}} + \frac{1}{n_{21}} + \frac{1}{n_{22}}}
\]

Exponentiate the resulting interval to obtain an interval for the OR
Some comments

- Notice that the sample and true odds ratios do not change if we transpose the rows and the columns.
- For both the OR and the RR, taking the logs helps with adherence to the error rate.
- Of course the interval for the log RR or log OR is obtained by taking:
  \[
  \text{Estimate} \pm Z_{1-\alpha/2} \times \text{SE}_{\text{Estimate}}
  \]
- Exponentiating yields an interval for the OR or RR.
- Though logging helps, these intervals still don’t perform altogether that well.
Example - RR

- For the relative risk, $\hat{p}_A = 11/20 = .55$, $\hat{p}_B = 5/20 = .25$
- $\hat{RR}_{A/B} = .55/.25 = 2.2$
- $\hat{SE}_{\log \hat{RR}_{A/B}} = \sqrt{\frac{1-.55}{.55\times20} + \frac{1-.25}{.25\times20}} = .44$
- Interval for the log RR: $\log(2.2) \pm 1.96 \times .44 = [-.07, 1.65]$
- Interval for the RR: $[.93, 5.21]$
Example - OR

- $\hat{OR}_{A/B} = \frac{11 \times 15}{9 \times 5} = 3.67$
- $\hat{SE}_{\log \hat{OR}_{A/B}} = \sqrt{\frac{1}{11} + \frac{1}{9} + \frac{1}{5} + \frac{1}{15}} = .68$
- Interval for log OR: $\log(3.67) \pm 1.96 \times .68 = [-.04, 2.64]$
- Interval for the OR: $[.96, 14.01]$
Example - RD

- For the risk difference
  \( \hat{RD}_{A-B} = \hat{p}_A - \hat{p}_B = .55 - .25 = .30 \)
- \( SE\hat{RD}_{A-B} = \sqrt{\frac{.55 \times .45}{20} + \frac{.25 \times .75}{20}} = .15 \)
- Interval: \( .30 \pm 1.96 \times .15 = [.15, .45] \)
Relative measures

The relative risk

The odds ratio
Relative measures

The relative risk

The odds ratio