Practice Problems (Confidence Intervals for Proportions)

- In a sample of patients from a certain population, 9 out of 10 individuals respond to the treatment.
  
  1. Show that \((0.5550; 0.9975)\) is a 95% confidence interval for the probability of response in that population. 
  
  2. Given the data, is there evidence against the assumption that the true response probability is 50% in the population? 
  
  3. What would the 95% confidence interval be if we had observed 10 responders among the 10 subjects sampled? 

Solution:

1. We have 
   
   \[
   P(X \geq 9|p = 0.5550) = P(X = 9|p = 0.5550) + P(X = 10|p = 0.5550) = \\
   \binom{10}{9} \times 0.5550^9 \times 0.4450^1 + \binom{10}{10} \times 0.5550^{10} \times 0.4450^0 = \\
   0.0222 + 0.0028 = 0.025,
   \]
   
   and 
   
   \[
   P(X \leq 9|p = 0.9975) = 1 - P(X = 10|p = 0.9975) = 1 - 0.9975^{10} = 0.025.
   \]
   
   Therefore, \((0.5550; 0.9975)\) is a 95% confidence interval for the probability of response in the population. 

2. There is evidence against the assumption that the true response probability in the population is 50%, as 0.5 is not contained in the 95% confidence interval. 

3. If we had observed 10 responders out of 10 subjects sampled, the 95% confidence interval would have been \((0.025^{1/10}; 1) = (0.69; 1)\). Note that the rule of thumb gives \(1 - 3/10 = 0.7\) for the lower bound.