Homework Assignment 4
Solutions

1. Please see the code.
   [ 2 points ]

2. See the code. The 99% confidence interval for the mean $\mu$ is ( 7.875 ; 9.117 ).
   [ 2 points ]

3. Please see the code. The difference between the sample means is 4.25. Our estimate of the standard error of the difference between the sample means is 9.33. In addition, we have an estimate of 15.56 for the "degrees of freedom" parameter we use for the t-distribution. The 97.5 percentile of the t distribution with 15.56 degrees of freedom is 2.12. The 95% confidence interval for the difference between the population means is ( -15.6 ; 24.1 ).
   [ 4 points ]

4. Please see the code. The 95% confidence interval for the treatment effect is ( 46.8 ; 152.8 ).
   [ 2 points ]

5. The standard error for sample mean will be $\sigma / \sqrt{n}$. Thus, the 95% confidence interval will have a margin of error of $1.96 \times \sigma / \sqrt{n}$. For this margin of error to be less than 1.5, we have
   
   \[
   1.96 \times \sigma / \sqrt{n} < 1 \iff \sqrt{n} > 1.96 \times \sigma / 1.5 \iff n > (1.96 \times \sigma / 1.5)^2 = (1.96 \times 4.5 / 1.5)^2 = 34.57.
   \]

   Therefore, a sample size of 35 is required.
   [ 3 points ]

6. (a) The standard error for the difference in sample means will be $\sqrt{\sigma^2/n + \sigma^2/n} = \sigma \sqrt{2/n}$. Thus, the 95% confidence interval will have a margin of error of $1.96 \times \sigma \sqrt{2/n}$. For this margin of error to be less than one, we have
   
   \[
   1.96 \times \sigma \sqrt{2/n} < 1 \iff \sqrt{n} > 1.96 \times \sigma \sqrt{2} \iff n > (1.96)^2 \times 2\sigma^2 = 3.84 \times 2 \times 100 = 768.32.
   \]

   Thus, we need 769 subjects per group!

   (b) Using the formula from class, the power is $\approx P \{ Z > 1.96 - (\Delta \sqrt{n})/(\sigma \sqrt{2}) \}$ where $Z$ is a standard Gaussian distribution. For 50% power, we thus need $1.96 - (\Delta \sqrt{n})/(\sigma \sqrt{2}) = 0$ since $P \{ Z > 0 \} = 0.5$. Solving this equation for $n$ yields
   
   \[
   \sqrt{n} > 1.96 \times \sigma \sqrt{2}/\Delta \iff n > (1.96)^2 \times 2 \times (\sigma/\Delta)^2 = 3.84 \times 2 \times 25 = 192.08.
   \]

   Thus, we need at least 193 subjects per group.
   [ 6 points ]
7. Please also see the code.

(a) You need 17 subjects per group, for a total of 34.
(b) The power is 56%.
(c) The power is up to 69% now.

[ 3 points ]