1. We have \( \bar{X} \sim \text{Normal}(\text{mean}=10, \text{SD} = \frac{2.5}{\sqrt{100}} = 0.25) \), approximately. Let \( Z = (\bar{X} - 10)/0.25 \).

   (a) \( \text{Pr}(\mid \bar{X} - 10 \mid < 0.1) = \text{Pr}(\mid Z \mid < 0.1/0.25) = \text{Pr}(\mid Z \mid < 0.4) \approx 31\% \).

   (b) \( \text{Pr}(\bar{X} > 10.25) = \text{Pr}(Z > 1) \approx 16\% \).

   [ 3 points ]

2. The confidence interval would be \( \bar{x} \pm 1.96 \times \sigma/\sqrt{n} \). This interval has width \( 2 \times 1.96 \times \sigma/\sqrt{n} \). Hence we need to find \( n \) such that

\[
2 \times 1.96 \times 0.75/\sqrt{n} \leq 1 \iff n \geq \left(2 \times 1.96 \times 0.75\right)^2 = 8.64.
\]

Thus we need to measure at least 9 replicates.

[ 3 points ]

3. We have \( n = 400, \bar{x} = 140, s = 25 \). Since \( n \) is very large, we can use a normal approximation, and the 95% confidence interval can be approximated by \( 140 \pm 1.96 \times 25/\sqrt{400} = 140 \pm 2.44 = (137.56; 142.44) \). The 99% confidence interval can be approximated by \( 140 \pm 2.58 \times 25/\sqrt{400} = 140 \pm 3.23 = (136.77; 143.23) \).

[ 3 points ]

4. Please also see the code.

   (a) For any single person we have

\[
P(X < 80) = P((X - 90)/5 < (80 - 90)/5) = P(Z < -2) = 0.0228,
\]

where \( X \sim N(\mu = 90, \sigma = 5) \) and \( Z \sim N(0,1) \). Thus, among 1,000 people randomly selected from this population, the expected number of people with FPG less than 80 mg/dl is \( n \times p = 1000 \times 0.0228 = 22.8 \), based on a Binomial distribution with \( n = 1000 \) and \( p = 0.0228 \).

   (b) For 25 people we have \( \text{E}[\bar{X}] = 90 \) and \( \text{Var}[\bar{X}] = \text{Var}[X]/25 = 1 \). Therefore

\[
P(\bar{X} > 92) = P((\bar{X} - 90)/1 > (92 - 90)/1) = P(Z > 2) = 0.0228.
\]

   (c) For any single person we have

\[
P(X > 100) = P((X - 90)/5 > (100 - 90)/5) = P(Z > 2) = 0.0228.
\]

Let \( Y \) be a Binomial distribution with \( n = 5 \) and \( p = 0.0228 \). Then

\[
P(Y \geq 4) = P(Y = 4) + P(Y = 5) = \binom{5}{4} \times 0.0228^4 \times (1 - 0.0228) + 0.0228^5 = 1.3 \times 10^{-6}.
\]
5. Please also see the code. We have $\bar{x}=101.8$, $s=11.6$, $n=10$, and thus, the estimated standard error is $s/\sqrt{n}=3.67$. For the critical value we use $qt(0.975, 9)$ which is 2.26. Thus, the 95% confidence interval can be approximated by $101.8 \pm 2.26 \times 3.67 = (93.49 ; 110.11)$. 

[2 points]

6. Please see the code. The 95% confidence interval for the population mean is (92.1;109.5), the 95% confidence interval for the population standard deviation is (5.2;20.3), and the 95% confidence interval for the population variance is (26.7;412.5). 

[4 points]