Homework Assignment 1
Solutions

1. (a) $\Pr(F_2 \text{ seed is smooth}) = \frac{3}{4} = 75\%$.
   (b) $\Pr(F_2 \text{ seed has genotype } Aa) = \frac{1}{2} = 50\%$.
   (c) $\Pr(F_2 \text{ seed has genotype } Aa \mid \text{ it is smooth}) = \frac{1}{2}/(3/4) \approx 67\%$.
   (d) $\Pr(F_3 \text{ seed is smooth } \mid F_2 \text{ has genotype } AA) = 100\%$.
   (e) $\Pr(F_3 \text{ seed is smooth } \mid F_2 \text{ has genotype } Aa) = \frac{3}{4} = 75\%$.
   (f) $\Pr(F_3 \text{ seed is smooth}) = \Pr(F_3 \text{ is smooth } \mid F_2 \text{ is AA}) \times \Pr(F_2 \text{ is AA}) + \Pr(F_3 \text{ is smooth } \mid F_2 \text{ is Aa}) \times \Pr(F_2 \text{ is Aa}) + \Pr(F_3 \text{ is smooth } \mid F_2 \text{ is aa}) \times \Pr(F_2 \text{ is aa}) = (1 \times (1/4)) + ((3/4) \times (1/2)) + (0 \times (1/4)) = 5/8 = 62.5\%$.
   (g) $\Pr(F_3 \text{ seed is smooth } \mid F_2 \text{ is smooth}) = \Pr(F_3 \text{ is smooth } \mid F_2 \text{ is AA}) \times \Pr(F_2 \text{ is AA } \mid F_2 \text{ is smooth}) + \Pr(F_3 \text{ is smooth } \mid F_2 \text{ is Aa}) \times \Pr(F_2 \text{ is Aa } \mid F_2 \text{ is smooth}) = (1 \times (1/3)) + ((3/4) \times (2/3)) = 5/6 \approx 83.3\%$.

2. $X$ should follow a Poisson($\lambda=2$) distribution (more precisely, it is a very good approximation).
   (a) $E(X) = 2$
   (b) $SD(X) = \sqrt{2} \approx 1.4$
   (c) $\Pr(X = 0) = e^{-2} \frac{2^0}{0!} = e^{-2} \approx 14\%$.
   (d) $\Pr(X = 5) = e^{-2} \frac{2^5}{5!} \approx 3.6\%$.
   (e) $\Pr(X > 2) = 1 - \Pr(X = 0) - \Pr(X = 1) - \Pr(X = 2) = 1 - e^{-2} - e^{-2} 2 - e^{-2} 2^2 / 2! \approx 32\%$.

3. Suppose $X \sim \text{Normal(mean=5,SD=3)}$. Let $Z = (X-5)/3$.
   (a) $\Pr(X < 6) = \Pr(Z < (6-5)/3) = \Pr(Z < 1/3) \approx 63\%$.
      In R, type `pnorm(6,5,3)` or `pnorm(1/3)`.
   (b) $\Pr(X > 0) = \Pr(Z > -5/3) = \Pr(Z < 5/3) \approx 95\%$.
      In R, type `1-pnorm(0,5,3)` or `pnorm(0,5,3,lower=F)` or `pnorm(5/3)`.
   (c) $\Pr(0 < X < 5) = \Pr(X > 0) - 1/2 \approx 45\%$.
      In R, type `pnorm(5,5,3) - pnorm(0,5,3)` or `pnorm(5/3) - 1/2`. 
(d) \( \Pr(2 < X < 8) = \Pr((2-5)/3 < Z < (8-5)/3) = \Pr(-1 < Z < 1) = 1 - 2 \times \Pr(Z < -1) \approx 68\% . \) 
In R, type `pnorm(8, 5, 3)-pnorm(2, 5, 3)` or `1-2*pnorm(-1)`.

(e) \( \Pr(|X - 5| > 2) = \Pr(|Z| > 2/3) = 2 \times \Pr(Z < -2/3) \approx 50\% . \)
In R, type `pnorm(7, 5, 3, lower=F)+pnorm(3, 5, 3)` or `2*pnorm(-2/3)`.

[ 4 points ]

4. (a) Let \( Z = (Y - 30)/5 \). Then \( E(Z) = 0 \) and \( SD(Z) = 1 \). 
(b) Let \( X = -Y \). Then \( E(X) = -E(Y) = -30 \), and \( SD(X) = SD(Y) = 5 \). 
(c) Let \( R = 5 + Y/3 \). Then \( E(R) = 5 + E(Y)/3 = 15 \), and \( SD(R) = SD(Y)/3 = 5/3 \). 
[ 2 points ]

5. Let \( n \) be the number of slides examined and \( X \) be the number that are positive. If the sample is positive, then \( X \sim \text{Binomial}(n, p=0.2) \). We seek \( n \) such that \( \Pr(X = 0) \leq 1\% \). We have
\[
\Pr(X = 0) = \binom{n}{0} \times p^0 \times (1-p)^{n-0} = (1-p)^n = 0.8^n .
\]
Thus, we need to solve the following equation: \( (0.8)^n \leq 0.01 \)
\[
0.8^n \leq 0.01 \iff \log(0.8^n) \leq \log(0.01) 
\iff n \log(0.8) \leq \log(0.01) 
\iff n(-0.223) \leq -4.605 
\iff n \geq 20.6
\]
Therefore, we must examine at least 21 slides. 
[ 4 points ]

6. See code. [ 3 points ]

7. See code. [ 3 points ]